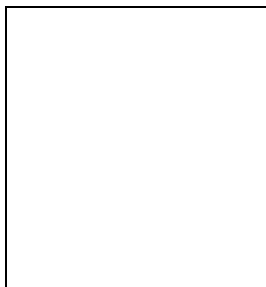


# EVOLUTION EQUATIONS IN THE ELECTROWEAK SECTOR OF THE STANDARD MODEL

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Energy-growing electroweak corrections in the Standard Model are potentially relevant for LHC physics, for Next generation of Linear Colliders (NLCs) and for ultrahigh energy cosmic rays. I present here the results of recent work in which electroweak evolution equations (the analogous of DGLAP equations in QCD) have been derived. The main features of these effects, mainly related to the fact that the electroweak sector is spontaneously broken, are pointed out.

## 1 Introduction

The study of radiative electroweak corrections in the Standard Model at very high (TeV scale) energies has taught us many interesting, mostly unexpected, features. Such corrections turn out to be huge, of the order of 10-20 % at one loop, and grow like the square of the log of the c.m. energy. This has triggered a fairly big amount of work on higher order corrections and on the possibility of resumming them<sup>1</sup>, particularly after the observation that such logarithms are tied to the infrared structure of the theory<sup>2</sup>. While resummation techniques can be mutuited from QCD, it has been clear from the beginning that the fact that the electroweak  $SU(2) \otimes U(1)$  sector is spontaneously broken causes important differences from an unbroken theory like the  $SU(3)$  strong interactions sector.

The most striking feature of electroweak radiative corrections at asymptotic energies is the following. Consider a very high energy process ( $\sqrt{s} \gg 100$  GeV) in which you include over all possible form of radiation in the final state: photons, gluons, but also Ws and Zs. Suppose that every kinematical variable defining the process is of the same order. A paradigm for this kind of observable is the process  $e^+e^- \rightarrow 2jets + X$  with  $s \approx |t| \approx |u|$ . Then, one would naively conclude that the cross section for this process depends on the only kinematical variable  $s$  and that its

asymptotic behaviour for large  $s$  is dictated by the renormalization group equations. However, this turns out to be false: even at the highest energies, fully inclusive observables depend on the weak scale  $M \approx 100$  GeV, which acts as an infrared cutoff for potentially divergent double logarithms. This dependence is due to a lack of cancellation between real and virtual emission, related to the fact that initial states are nonabelian (isospin) charges<sup>3</sup>.

In this note I present the results of two papers in which the infrared evolution equations in the Standard Model of electroweak interactions have been derived. These equations correspond to the usual Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations in QCD, however because of symmetry breaking in the electroweak sector a number of differences emerge.

- because of the above mentioned lack of cancellation, electroweak evolution equations take into account not only single logarithmic collinear contributions  $\sim \alpha_W \log \frac{s}{M^2}$ , but also double logarithms of infrared/collinear origin  $\sim \alpha_W \log^2 \frac{s}{M^2}$ .
- new splitting functions, that are different from the ones appearing in QCD, have to be introduced.
- while kinematics closely resembles that of QCD, the lack of isospin averaging (compared with colour averaging in QCD) render the isospin structure much more complicated.

Energy-growing electroweak corrections in the Standard Model are potentially relevant for LHC physics<sup>4</sup>, for Next generation of Linear Colliders (NLCs)<sup>5</sup> and for ultrahigh energy cosmic rays<sup>6</sup>.

## 2 Left fermions in the $g' \rightarrow 0$ limit

In this section I consider lepton initiated Drell Yan process of type  $e^+(p_1) e(p_2) \rightarrow q(k_1)\bar{q}(k_2)+X^*$  where  $s = 2p_1 \cdot p_2$  is the total invariant mass and  $Q^2 = 2k_1 \cdot k_2$  is the hard scale. I consider double log corrections in relation to the SU(2) electroweak gauge group, i.e. I work in the limit where the U(1) coupling  $g'$  is zero. This process has been analyzed in<sup>7,8</sup>, to which I refer for details; I consider it here as a simple example of how evolution equations can be derived. The general formalism used to study electroweak evolution equations for inclusive observables has been set up in<sup>9</sup>; I summarize it here briefly. To begin with, by arguments of unitarity, final state radiation can be neglected when considering inclusive cross sections<sup>7</sup>. Then one is led to consider the dressing of the overlap matrix  $\mathcal{O}_{\alpha\beta}^{\alpha'\beta'} = \langle \beta\beta' | S^+ S | \alpha\alpha' \rangle$ ,  $S$  being the  $S$ -matrix, where only initial states indices appear explicitly (see fig. 1).

At the leading level, all order resummation in the soft-collinear region is obtained by a simple expression that involves the t-channel total isospin  $T$  that couples indices  $\alpha, \beta$ :

$$\mathcal{O}^H \rightarrow \mathcal{O}^{resummed} = e^{-\frac{\alpha_W}{4\pi} [T(T+1)] \log^2 \frac{s}{M_w^2}} \mathcal{O}^H \quad (1)$$

$\alpha_W$  being the weak coupling,  $\mathcal{O}_H$  the hard overlap matrix written in terms of the tree level  $S$ -matrix and  $M_w \sim M_z$ .

At subleading order, the dressing by soft and/or collinear radiation is described at all orders by infrared evolution equations, that are  $T$ -diagonal as far as fermions and transverse gauge bosons are concerned<sup>9</sup>. In order to write down the evolution equations for the case of initial left fermions, we first consider one loop corrections. At the one loop level, virtual and real corrections in NLL approximation can be written as:

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\*note that the process considered, like all others in this paper, is fully inclusive, meaning also  $W, Z$  radiation is included.

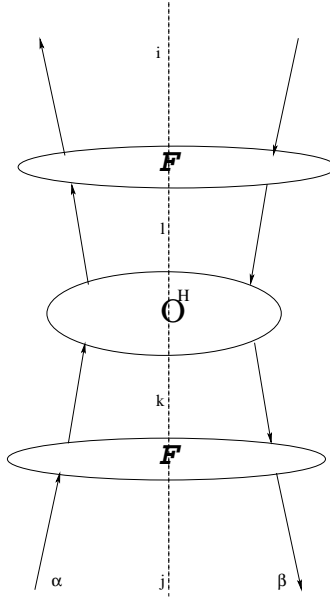


Figure 1: Graphical picture of the factorization formula 3.)

$$\delta \mathcal{O}_{L\alpha\beta} = \frac{\alpha_W}{2\pi} \int_{M^2}^s \frac{dk_\perp^2}{k_\perp^2} \int_0^1 \frac{dz}{z} \left\{ P_{ff}^R(z) \theta(1-z - \frac{k_\perp}{\sqrt{s}}) t_{\beta\beta'}^A t_{\alpha'\alpha}^A \mathcal{O}_{\alpha'\beta'}^H(zp) + \right. \\ \left. P_{gf}^R(z) [t^B t^A]_{\beta\alpha} \mathcal{O}_{AB}^H(zp) + C_f P_{ff}^V(z, \frac{k_\perp}{\sqrt{s}}) \mathcal{O}_{\alpha\beta}^H(p) \right\} \quad (2)$$

where  $P_{ff}^R(z)$ ,  $P_{gf}^R(z)$  and  $P_{ff}^V(z, k_\perp)$  are defined in <sup>10</sup>. The indices below the overlap matrix label the kind of particle:  $L$  = Left fermion and  $g$  = gauge boson ; indices  $\alpha, \beta$  refer to the isospin index ( $\alpha = 1$  corresponds to  $\nu$ ,  $\alpha = 2$  to  $e$ ) of the lower legs while upper legs indices are omitted.

The one loop formula (2) is consistent with a general factorization formula of type (see fig. 1))

$$\mathcal{O}_j^i(p_1, p_2; k_1, k_2) = \int \frac{dz_1}{z_1} \frac{dz_2}{z_2} \sum_{k,l}^{L,R,g} \mathcal{F}_j^k(z_1; s, M^2) \mathcal{O}_k^H(z_1 p_1, z_2 p_2; k_1, k_2) \mathcal{F}_l^i(z_2; s, M^2) \quad (3)$$

where  $i, j$  label the kind of particle ( $L$ =left fermion,  $g$ =gauge boson), and where isospin flavor indices in the overlap function  $\mathcal{O}$  and structure function  $\mathcal{F}$  are understood.

If the factorization formula (3) is assumed to be valid at higher orders as well, the structure functions will satisfy evolution equations with respect to an infrared-collinear cutoff  $\mu$  parameterizing the lowest value of  $k_\perp$ , as follows ( $t = \log \mu^2$ ):

$$-\frac{\partial}{\partial t} \mathcal{F}_j^i \alpha\beta = \frac{\alpha_W}{2\pi} \left\{ C_f \mathcal{F}_j^i \alpha\beta \otimes P_{ff}^V + [t^C \mathcal{F}_j^i t^C]_{\alpha\beta} \otimes P_{ff}^R + [t^B t^A]_{\beta\alpha} \mathcal{F}_g^i \otimes P_{gf}^R \right\} \quad (4)$$

In these equations  $t^A$  denote the isospin matrices in the fundamental representation and  $\mathcal{F}_j^i \alpha\beta$  denotes the distribution of a particle  $i$  (whose isospin indices are omitted) inside particle  $j$  (with isospin leg indices  $\alpha, \beta$ ).  $\mathcal{F}_j^i t$  is the transpose matrix  $\mathcal{F}_j^i \beta\alpha$ . Furthermore, we have defined the convolution  $[f \otimes P](x) \equiv \int_x^1 P(z) f(\frac{x}{z}) \frac{dz}{z}$ . Since the index  $i$  is always kept fixed in (4), it can be

omitted, with the understanding that, for instance,  $\mathcal{F}_j$  collectively denotes all  $\mathcal{F}_j^i$  with any value of  $i$ .

Eqn. (4) is a matricial evolution equation; in order to make it useful the corresponding scalar equations must be written. This can be done by exploiting the  $SU(2)_L$  symmetry which allows to classify the states according to their isospin quantum numbers. By coupling the lower legs  $\alpha, \beta$  in fig. 1 one obtains the t-channel isospin eigenstates:

$$|\mathbf{T} = 0\rangle = \frac{1}{\sqrt{2}}(|\nu\nu^*\rangle + |ee^*\rangle) \quad |\mathbf{T} = 1\rangle = \frac{1}{\sqrt{2}}(|\nu\nu^*\rangle - |ee^*\rangle) \quad (5)$$

which have  $T_L^3 = 0$  since cross sections always have a given particle on leg  $\alpha$  and its own antiparticle on leg  $\beta$ . I now project the structure operators  $\mathcal{F}$  on these states, omitting the upper leg indices:

$$\begin{aligned} f_L^{(0)} &= \frac{\langle \nu\nu^* + ee^* | \mathcal{F}_L | \rangle}{2} = \frac{\mathcal{F}_{\nu\nu} + \mathcal{F}_{ee}}{2} = \frac{1}{2} \text{Tr} [\mathcal{F}_L] \\ f_L^{(1)} &= \frac{\langle \nu\nu^* - ee^* | \mathcal{F}_L | \rangle}{2} = \frac{\mathcal{F}_{\nu\nu} - \mathcal{F}_{ee}}{2} = \text{Tr} [t^3 \mathcal{F}_L] \end{aligned} \quad (6)$$

Last step in eqs. (6) represents a convenient way to extract the scalar coefficients  $f_j^{(T)}$  from  $\mathcal{F}_j$ ; namely, by taking appropriate traces with respect to the soft leg  $j$ . For instance  $f_L^{(0)}$  corresponds to  $\frac{1}{2}(\mathcal{F}_{ee} + \mathcal{F}_{\nu\nu})^9$  and can be obtained by  $\text{Tr}_j[\mathcal{F}]$ ; here and in the following the trace is taken with respect to the indices of the soft lower scale leg  $j$ . Notice that since gauge and mass eigenstates do not necessarily coincide, one has to introduce “mixed legs” with particles belonging to different gauge representations on leg  $\alpha$  and  $\beta$ . We label these cases by  $i = LR$  for the mixed left/right fermion leg,  $i = B3$  for the mixed  $W_3 - B$  gauge bosons and  $i = h3$  for the Higgs sector case. These mixing phenomena are interesting by themselves and have been considered in<sup>8,11,12,13</sup> at double log level.

Projecting eq. (4) for instance on the  $T = 0$  component we obtain:

$$-\frac{\partial}{\partial t} \text{Tr}[\mathcal{F}_f] = \frac{\alpha_W}{2\pi} \left\{ C_f \text{Tr}[\mathcal{F}_f] \otimes P_{ff}^V + \text{Tr}[t^C \mathcal{F}_f^t t^C] \otimes P_{ff}^R + \text{Tr}[t^B t^A] \mathcal{F}_{AB} \otimes P_{gf}^R \right\} \quad (7)$$

where the traces are taken, here and in the following, with respect to the soft leg indices. This gives:

$$-\frac{\partial}{\partial t} f_L^{(0)} = \frac{\alpha_W}{2\pi} \left( \frac{3}{4} f_L^{(0)} \otimes (P_{ff}^V + P_{ff}^R) + \frac{3}{4} f_L^{(0)} \otimes P_{gf}^R \right) \quad (8)$$

after taking into account that  $\text{Tr}[t^B t^A] \mathcal{F}_{AB} = \frac{1}{2} \sum_A \mathcal{F}_{AA} = \frac{1}{2} \text{Tr}[\mathcal{F}]$  and that  $f_L^{(0)} = \frac{1}{3} \text{Tr}[\mathcal{F}_L]$ .

Finally, the last step in obtaining the all order resummed overlap matrix, requires the evolution of the  $f_i^{(T)}$ 's according to eqn. (8) with appropriate initial conditions, and inserting the evolved  $f_i^{(T)}$ 's into (3). This can be done by exploiting the recovered isospin symmetry, which allows us to write:

$$\mathcal{O}_j^i(p_1, p_2; k_1, k_2) = \sum_T \int \frac{dz_1}{z_1} \frac{dz_2}{z_2} \sum_{k,l}^{L,R,g} f_j^{(T)}(z_1; s, M^2) \mathcal{O}_k^H(T)(z_1 p_1, z_2 p_2; k_1, k_2) f_l^i(T)(z_2; s, M^2) \quad (9)$$

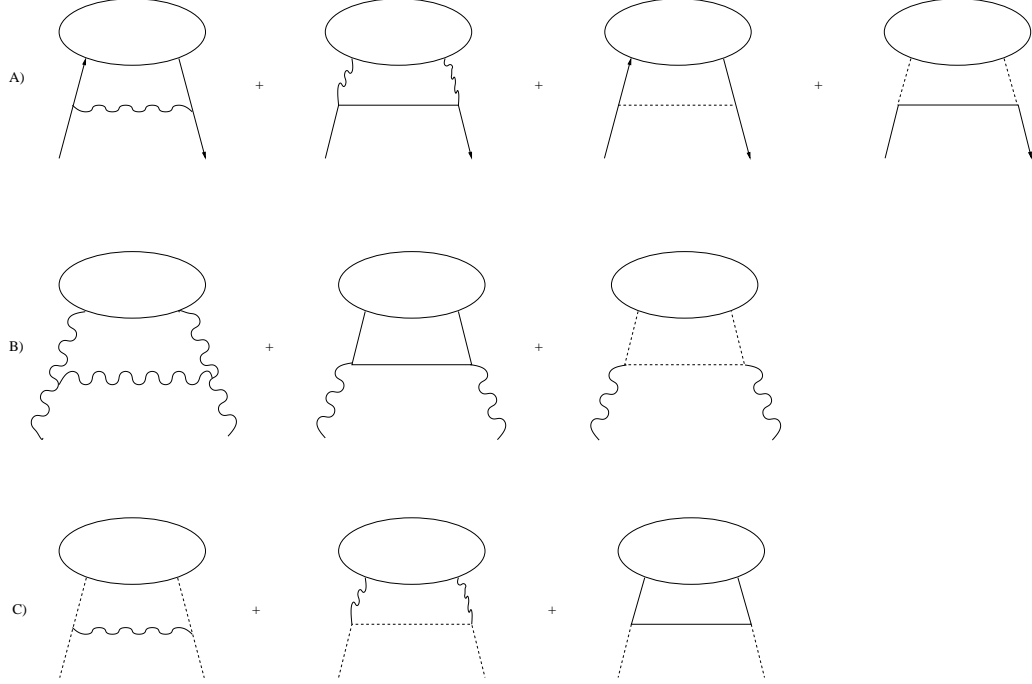


Figure 2: Leading real emission Feynman diagrams in axial gauge: A) Feynman diagrams contributing to the evolution of the fermionic structure functions; B) Feynman diagrams contributing to the evolution of the transverse gauge boson structure functions; C) Feynman diagrams contributing to the evolution of the scalar structure functions. The wavy lines are transverse gauge bosons, dashed lines stay for Higgs sector particles and straight lines for fermions.

### 3 Electroweak evolution equation in the full Standard Model

The set of complete infrared evolution equation in the electroweak sector of the Standard Model have been derived in<sup>10</sup>. According to the equivalence theorem, we replace longitudinal gauge bosons with the corresponding Goldstone bosons. We choose to work in an axial gauge so that this substitution can be done without higher order corrections in the definition of the asymptotic states<sup>14</sup>. The procedure of writing down evolution equations for the matricial  $\mathcal{F}_{\alpha'\beta'}^{\alpha\beta}$  and then extracting the corresponding equations for the scalar components  $f_i^j$  is similar to the one outlined in previous section. However a number of contributions has to be added: the ones proportional to  $g'$ , and the ones proportional to the third generation Yukawa couplings, which cannot be neglected even in the high energy limit. While the various contributions are summarized in fig. I refer to<sup>10</sup> for the complete set of equations. A further complication is due to the fact that, in order to provide a complete classification of the states, the conserved quantum number  $CP$  is needed on top of the  $SU(2) \otimes U(1)$  quantum numbers.

Of course, the task of writing evolution equations for the full Standard Model is still incomplete. Having in mind hadronic initial states like in the LHC for instance, one has to add QED effects at low ( $\mu < M$ ) scale, and QCD effects. Moreover, a careful treatment of matching conditions at the weak scale  $M$  is mandatory. These will be the key points of our future research line.

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